2: Newton’s Apple

Physics 17: Black Holes and Extreme Astrophysics

Homework

- Available after class today. Due before class next Thursday. Marked work back after class a week later
- You can download the problems from Canvas under Assignments
- Submit your answers as a PDF file
  - Type your answers
  - Write your answers on paper and scan or photograph (save image as PDF)
- Upload at gradescope.com
  - Log in with your Stanford ID. You should be enrolled in Physics 17 and see the first homework assignment
- Email dan.wilkins@stanford.edu if you have problems with Gradescope

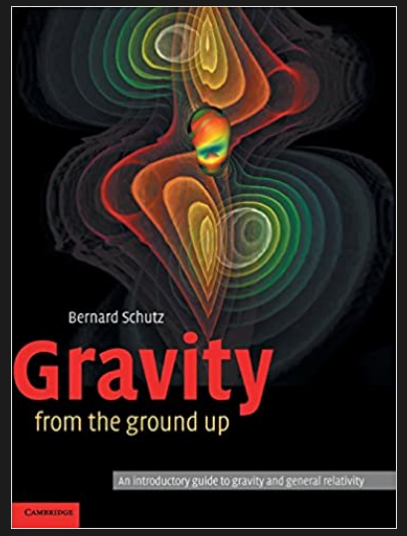
Office Hours

Go to pollev.com/wilkins

Goals

- Introduce force, dynamics and Newton’s Laws of Motion
- Describe the law of Gravitation
- Apply the law of Gravitation on Earth and in space
- Introduce the Principle of Equivalence

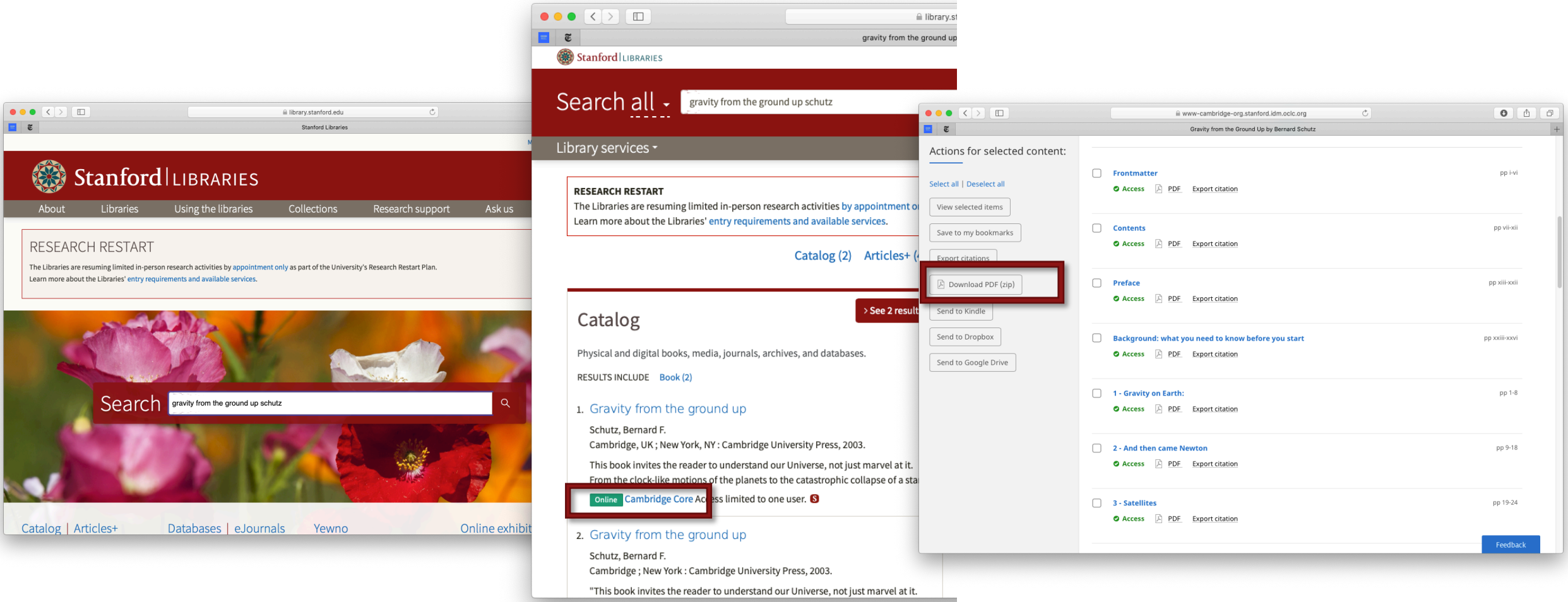
Reading



Schutz

- Chapter 2: Newton's laws, equivalence principle (p9-18)
- Chapter 4: Orbits and the Solar system (p25-27, 31-33)

You can access this book online via library.stanford.edu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pp i-vi
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Contents
pp vii-xii
Preface
pp xiii-xxii
Background: what you need to know before you start
pp xxiii-xxvi
1 - Gravity on Earth:
pp 1-8
2 - And then came Newton
pp 9-18
3 - Satellites
pp 19-24
Feedback

Motion in a Circle

- The speed of the ball is constant, but it's velocity changes (velocity has a direction) so it stays tangent to the circle
- Therefore, the ball accelerates, caused by the force of tension in the string:

$$\Delta v = a\Delta t$$

- Think about how the ball moves in a very short time, Δt, we can draw a triangle. It moves a distance:

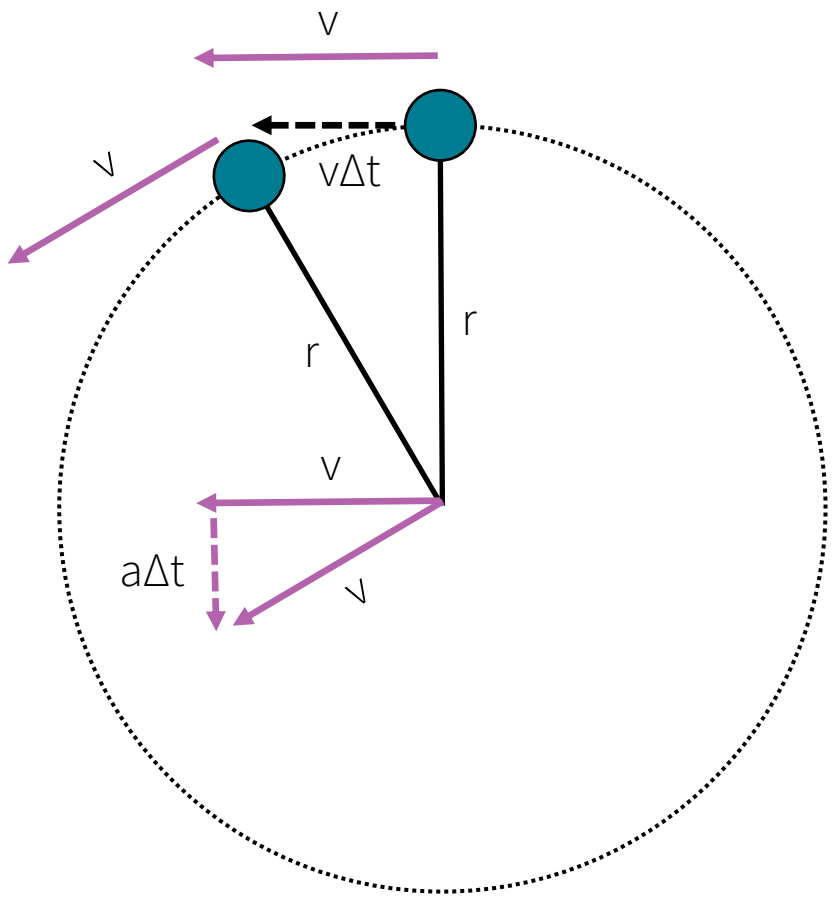
$$d = v\Delta t$$

- We can draw a second triangle of the velocities. The angle between them is the same, so they are *similar triangles*. This means that

$$\frac{a\Delta t}{v} = \frac{v\Delta t}{r}$$ TMI

- Therefore, the acceleration, towards the center of the circle

$$a = \frac{v^2}{r}$$ MaT



MaT

Motion in a Circle

- The angular velocity is the angle the ball travels round the circle per time t:

$$\omega = \frac{\theta}{t}$$

- The distance travelled around a little bit of the circle, $d = \theta r$, so the linear velocity (the speed the ball is travelling at):

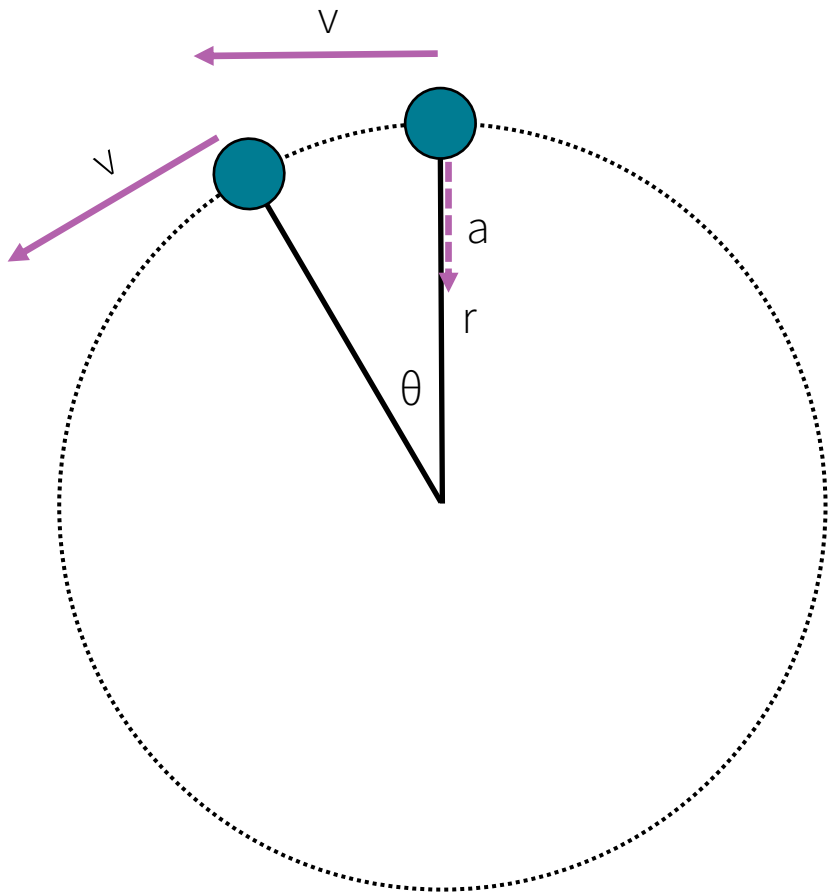
$$v = \omega r$$

- The total distance the ball has to travel to complete one circuit is the circumference, $D = 2\pi r$, so the period (the time taken to complete one circuit)

$$P = \frac{2\pi}{\omega} \text{ or } P = \frac{2\pi r}{v}$$

- The ball is always accelerating towards the center of the circle

$$a = \frac{v^2}{r} \text{ or } a = \omega^2 r$$



QaD

Hubble Space Telescope

Hubble orbits the Earth at an altitude:

$$h = 550\text{km}$$

The radius of the Earth is ~6300km, so the radius of Hubble's orbit is

$$r = 6300 + 550 = 6850\ \text{km}$$

It completes one orbit in a time period:

$$P = 95\ \text{min} = 5{,}700\ \text{s}$$

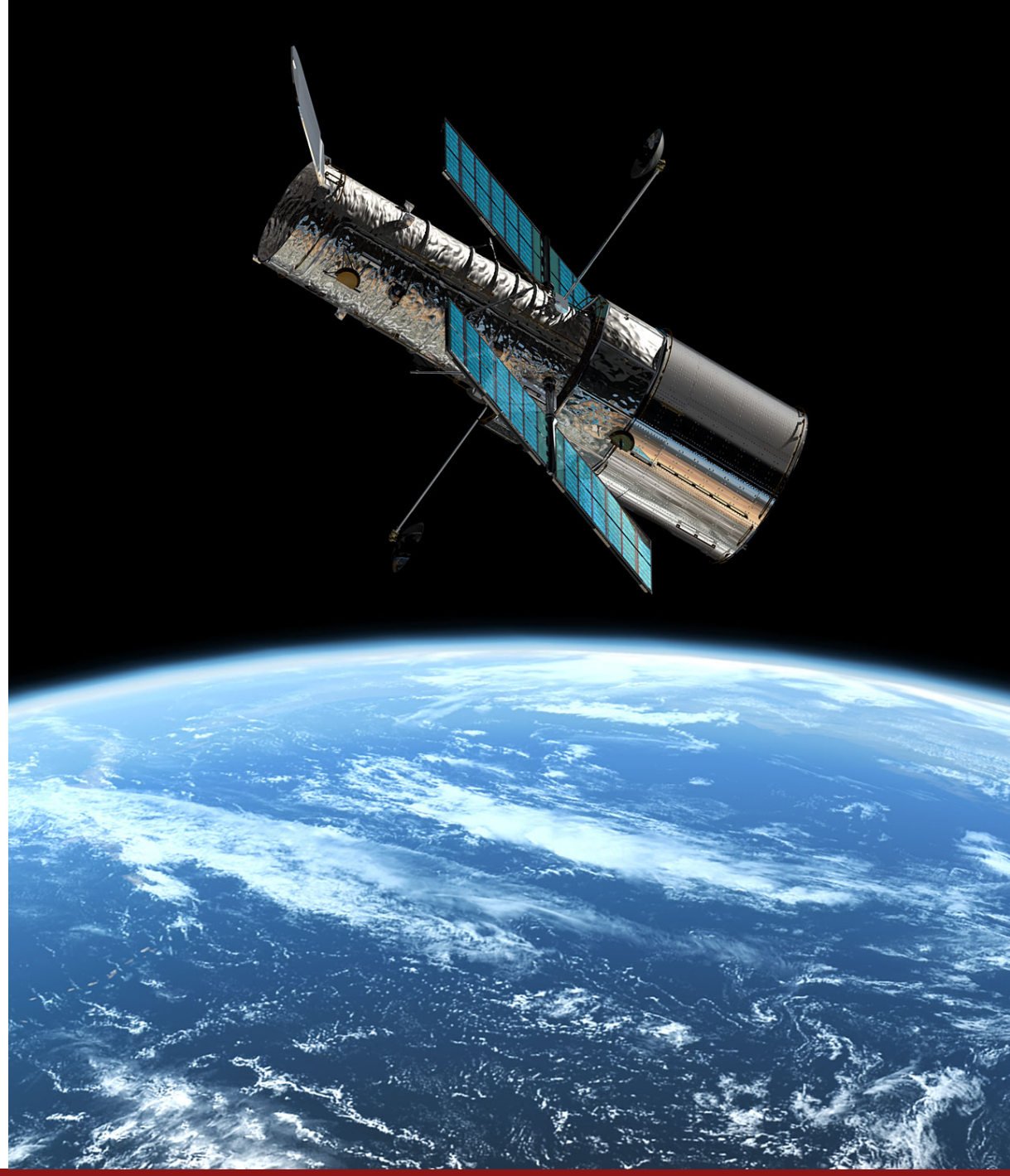
Therefore, its speed:

$$v = \frac{2\pi r}{P} \approx 8\ \text{km s}^{-1}$$

And acceleration:

$$a = \frac{v^2}{r} \approx 9\ \text{m s}^{-2} \approx g$$

Close to the Earth, acceleration due to gravity is approx. constant



QaD

The Moon

Distance from Earth:

r = 400,000km

Period

P = 27 days

Therefore, its speed:

$$v = \frac{2\pi r}{P} \approx 1\ \mathrm{km\ s^{-1}}$$

And acceleration:

$$a = \frac{v^2}{r} \approx \frac{1}{400}\ \mathrm{m\ s^{-2}} \approx \frac{g}{4000}$$

QaD

The Earth

Distance from Earth to Sun:

r ~ 150,000,000km = 1 AU (“Astronomical Unit”)

Period

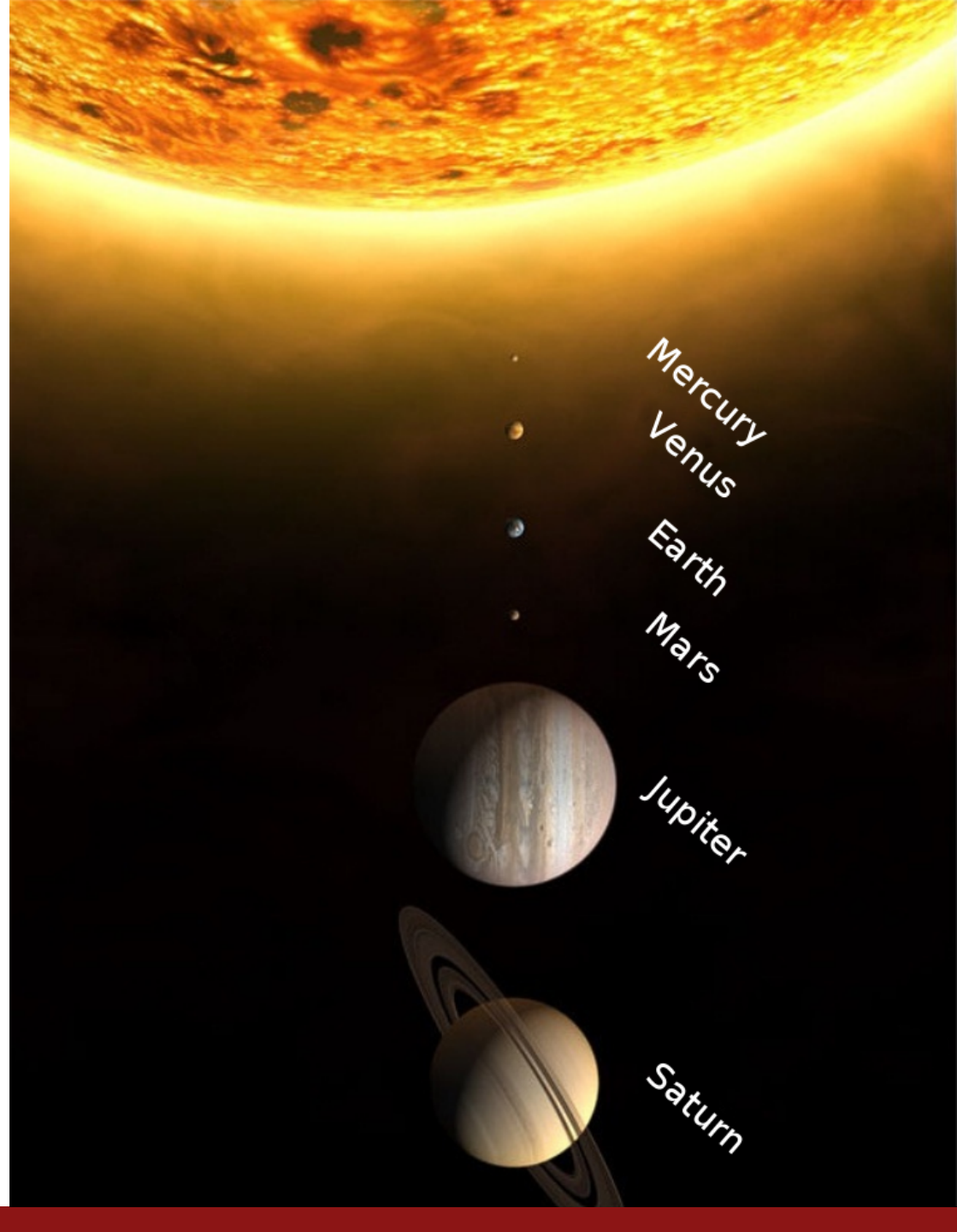
P = 1 year ~ 30,000,000 s

Therefore, its speed:

$$v = \frac{2\pi r}{P} \approx 30\ \mathrm{km\ s^{-1}}$$

And acceleration:

$$a = \frac{v^2}{r} \approx 10\ \mathrm{mm\ s^{-2}} \approx \frac{g}{1000}$$



Exponential Notation

MaT

For writing down big or small numbers…

$100 = 10^2$ | $1/100 = 0.01 = 10^{-2}$

$1000 = 10^3$ | $1/1000 = 0.001 = 10^{-3}$

$1000000 = 10^6$ | $1/1000000 = 0.000001 = 10^{-6}$

Scientific notation…

$300 = 3 \times 10^2$ | $0.05 = 5 \times 10^{-2}$

$4{,}500 = 4.5 \times 10^3$ | $0.00101 = 1.01^{-3}$

$2{,}010{,}000 = 2.01 \times 10^6$ | $0.000000055 = 5.5^{-8}$

Multiplication and division made easy…

$10^6 \times 10^9 = 10^{6+9} = 10^{15}$ | $10^6 / 10^9 = 106^{-9} = 10^{-3}$

$3x10^6 \times 5x10^9 = (3x5) \times 10^{6+9} = 1.5 \times 10^{16}$ | $6x10^6 / 2x10^4 = (6/2) \times 10^{6-4} = 3 \times 10^2$

SI notation

1km (1 kilometer) = 10^3 m | 1cm (1 kilometer) = 10^{-2} m

1Mm (1 megameter) = 10^6 m | 1mm (1 milimeter) = 10^{-3} m

1Gm (1 gigameter) = 10^9 m | 1µm (1 micrometer) = 10^{-6} m

Some numbers to get your head around...!

- Approx. $10^{20} \sim 10^{24}$ grains of sand on the Earth
- 10^{12} galaxies in the (visible) Universe (at least!)
- 10^{24} stars in the Universe

QaD

The Stars in the Galaxy

Stars in the outer arms travel at:

$$v \sim 300 \text{ km s}^{-1}$$

Distance from center to outer arms:

$$r = 3 \times 10^{20} \text{ m}$$

Therefore, the period of time for the Sun to complete one orbit:

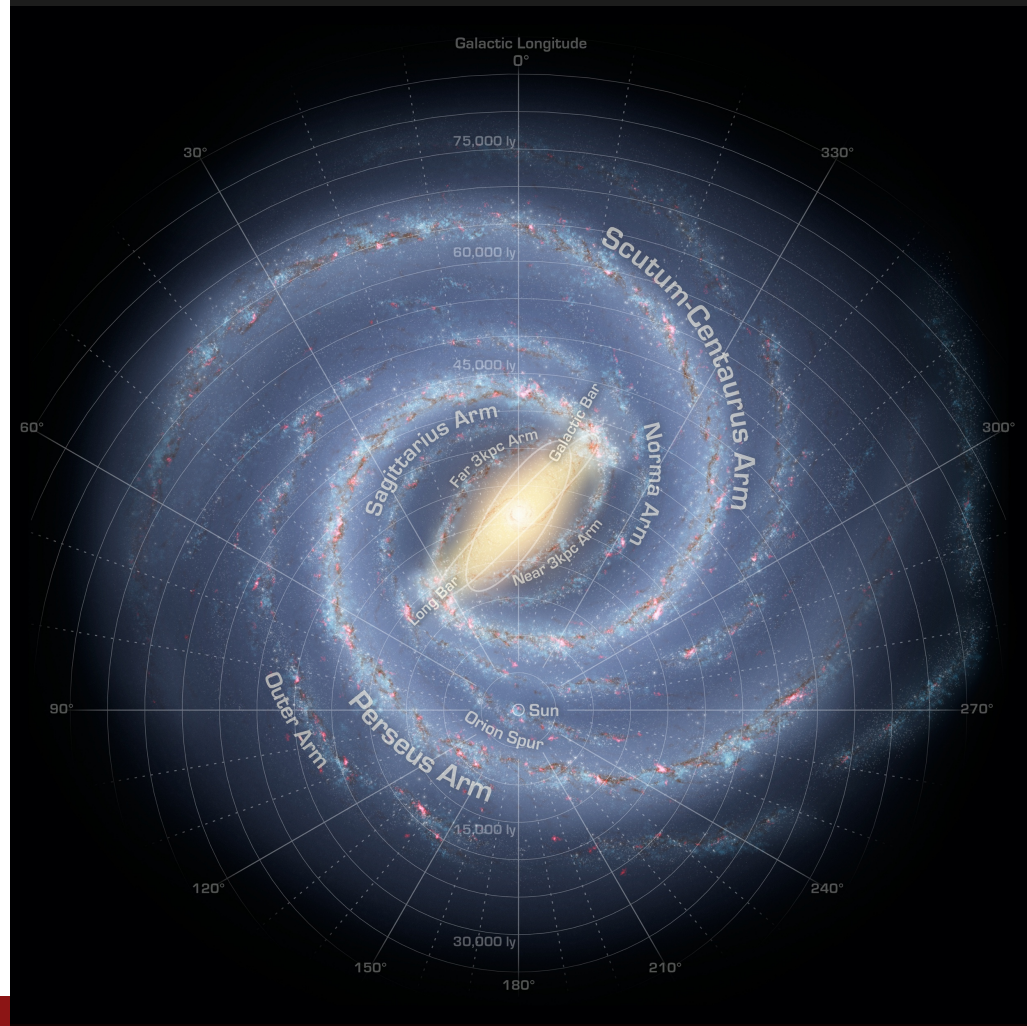
$$P = \frac{2\pi r}{v} \approx 2 \times 10^{8} \text{ yr}$$

And acceleration:

$$a = \frac{v^2}{r} \approx 3 \times 10^{-10} \text{ m s}^{-2} \approx 10^{-11} g$$

We are in the Milky Way, a spiral galaxy. Our Sun is one of about 100,000,000,000 stars

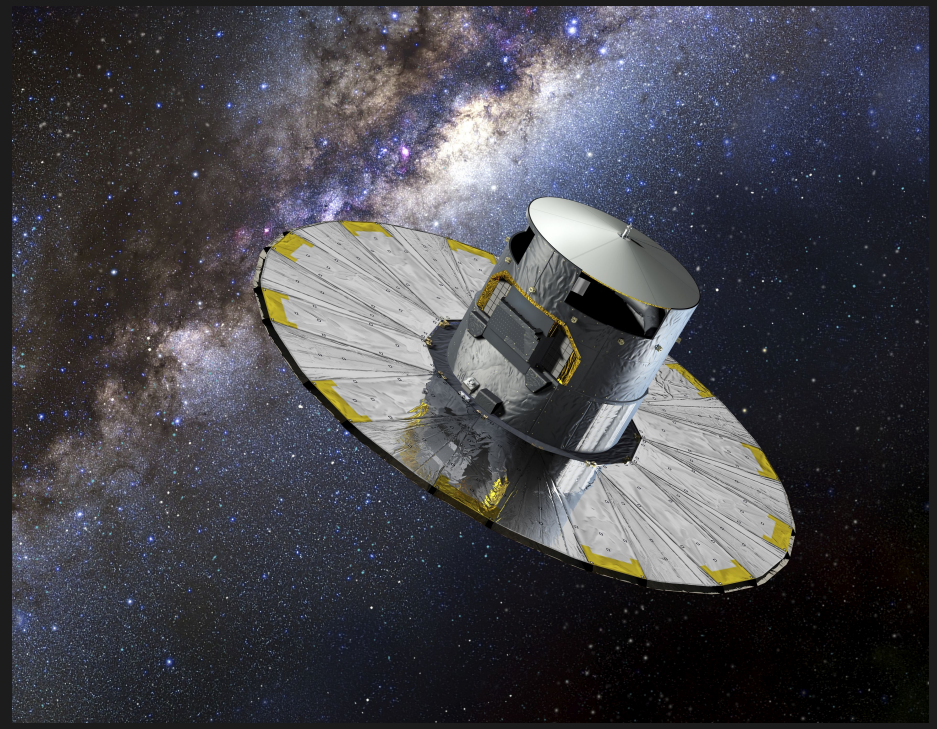
The stars travel in orbit around the center of the galaxy



The Dance of the Stars

ESA’s Gaia satellite is accurately mapping the positions and measuring the (extremely slight) motion

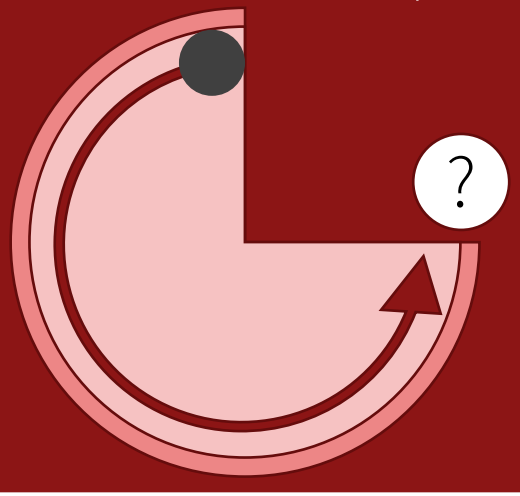
Based on these measurements, we can predict where the stars will be in the sky millions of years from today



Q1:

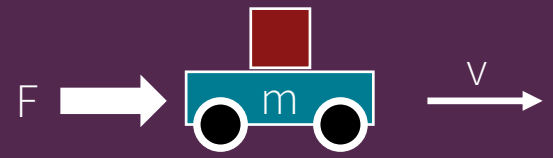
If we throw a ball in outer space, what will happen to its speed and direction as it travels?

Now, we roll the ball around the outer rim of a plate so that it travels round the circle. A quarter of the plate is missing, which direction will the ball go when it gets to the end of the plate?

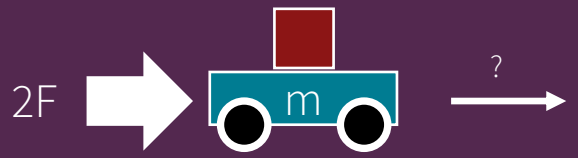


Q2:

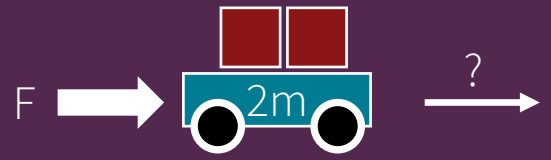
We push a cart with a force (F) for some amount of time (t) so that it starts moving and reaches a certain speed (v)



What speed does it reach if I push the cart with twice the force (2F)?



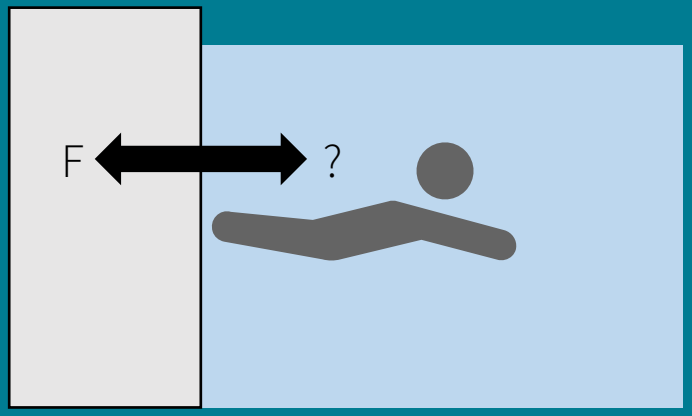
How about if the cart is twice as heavy?



What if I push it up a hill?

Q3:

A swimmer kicks off the side of the pool, applying a force F to the wall?



What is the force that accelerates the swimmer?

Newton's First Law

Objects at rest remain at rest, or objects in motion continue to move in a straight line, at a constant velocity unless a force is applied

A force is a changer of velocity

Newton's Second Law

The total force (F) acting on an object (required to give it acceleration a), is equal to its mass (m) multiplied by its acceleration (a)

$$F = ma$$

Newton's Third Law

"For every action there is an equal and opposite reaction"

When an object exerts a force on another object, that other object simultaneously exerts an equal force, in the opposite direction, back on the first object

Bigl

Mass, Force & Momentum

- Mass is the quantity of matter in an object
  - Mass is conserved (according to Newton)
- Momentum is mass x velocity ($p = mv$)
  - Has a magnitude and direction
  - Momentum is conserved (e.g. during a collision or a rocket launch)
- Force is rate of change of momentum
- Weight is the gravitational force (c.f. mass)

Momentum is always conserved, e.g. during a collision

The total momentum before the collision is equal to the total momentum after the collision (remembering that momentum is directed, so if the motion is backwards, the momentum is negative)

Before

After

$$m_1u_1 + m_2u_2 = -m_1v_1 - m_2v_2$$

The change in momentum of a rocket is equal to the momentum of the fuel it ejects

Q1:

We drop a ping pong ball (mass 3g or 0.1oz) and a 16lb (7kg) bowling ball from the top of a tower.

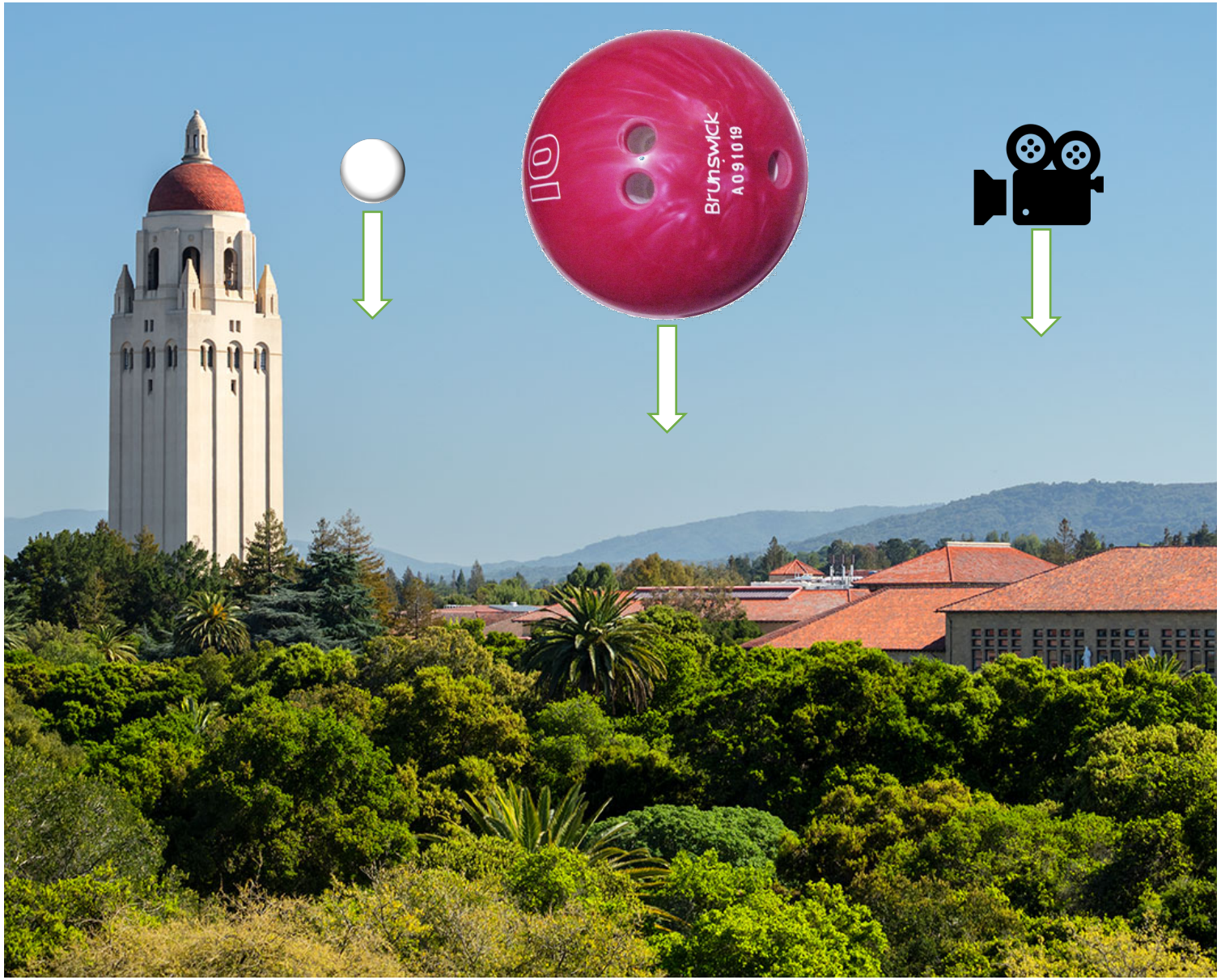
Which one hits the ground first?

Q2:

We drop a camera off the tower at the same time as a ball.

How does the ball appear to move in the video?

What if we throw the ball sideways off the tower?



Q1

Both the ping pong ball and the bowling ball accelerate at the same rate and hit the ground at the same time

$g \sim 9.8\ m/s^2$

Gravitational acceleration does not depend on the mass of the object

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Q2a

The camera accelerates downwards with the ball

In the camera's view, ball appears to be stationary

Q1

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$g \sim 9.8\ m/s^2$

Gravitational acceleration does not depend on the mass of the object

Q2a

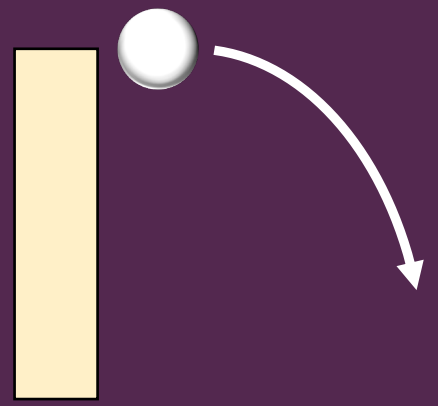
The camera accelerates downwards with the ball

In the camera's view, ball appears to be stationary

Q2b

As seen from the tower or the ground:

- The ball accelerates vertically at rate $g \sim 9.8\ m/s^2$
- The horizontal component of the ball's velocity remains constant (the same velocity we threw it at off the tower)
- The ball follows a curved path to the ground



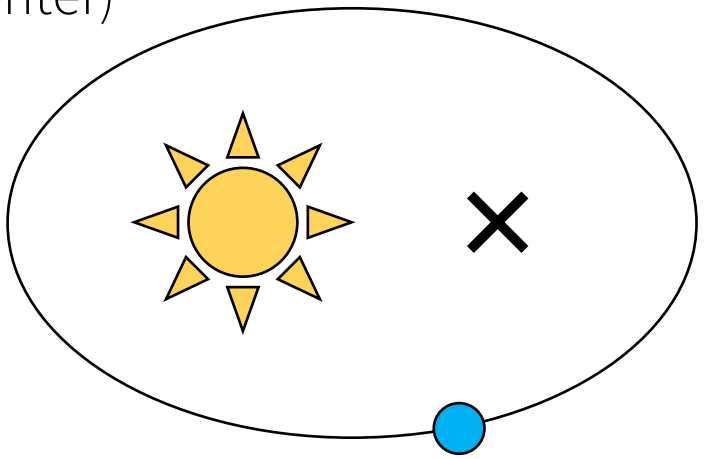
As seen by the camera:

- The camera accelerates vertically with the ball
- The horizontal component of the ball's velocity remains constant
- The ball moves in a straight line, horizontally, at a constant speed, as if there is no force
- According to the freely falling camera, there is no gravity

WILtY

Motion of the Planets

- Kepler (1571-1630) studied the motion of the planets using observations of how their positions in the sky move made by Tycho Brahe
- Orbits are ellipses (almost circular), Sun is at one focus (approx. center)



- The time taken to complete one orbit (the period)

$P \sim r^{\frac{3}{2}}$ MaT

QaD

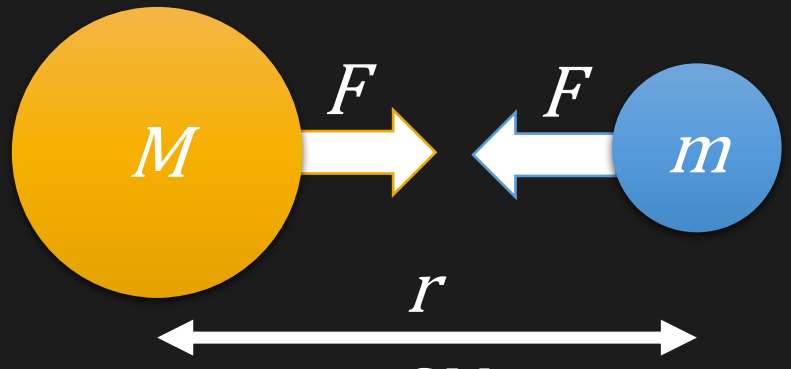
Planet	Distance to Sun	Period		
Earth	~ 1 AU			1yr
Jupiter	~ 5 AU	$5^{3/2}$	=	12yr
Pluto	~ 40 AU	$40^{3/2}$	=	250yr

*an Astronomical Unit (AU) is the distance from the Earth to the Sun, approx. 150 million km (93 million miles)

Bigl

Newton's Universal Law of Gravitation

- Isaac Newton (1665-87) – is the force that makes an apple fall to Earth the same that keeps the planets in orbit around the Sun?
- Started from Kepler's Laws (distance vs. period) and discovered that the gravitational force must fall as the distance squared $^1/_{r^2}$ MaT
- With this sort of force, planets follow an ellipse
- Gravity is the mutual attraction between all massive objects. Force increases in proportion to the mass of each object



$$F = \frac{GMm}{r^2}$$

Acceleration does not depend on mass of the falling object (m cancels):

$$ma = \frac{GMm}{r^2}$$

The gravitational constant:

$$G = 6.67\times10^{-11}\ \mathrm{N\ m^2 kg^{-2}}$$

Gravity is a weak force! We only really notice gravity from an object with a large mass (e.g. the Earth, $M_\oplus = 6\times10^{24}$ kg, or the Sun, $M_\odot = 2\times10^{30}$kg)

Motion of the Galaxies

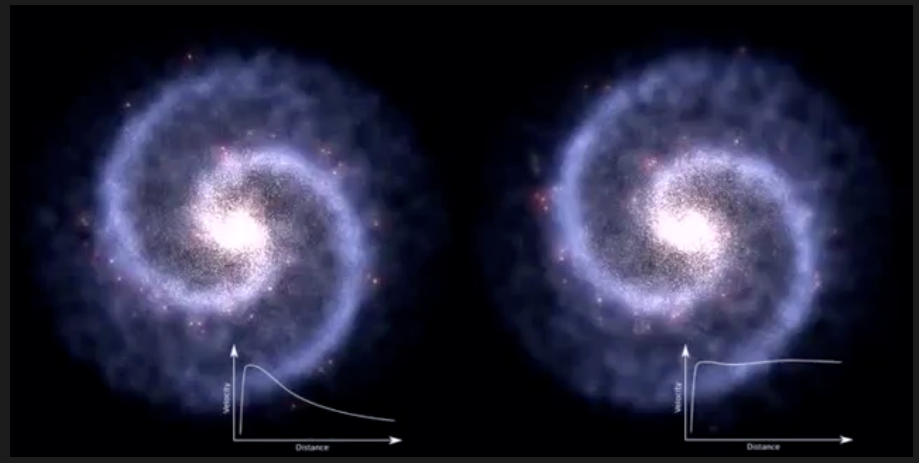
- Stars orbit around the center of a galaxy
- They are held in orbit by the gravitational attraction of all the stars inside their orbit
- Force required to maintain motion in a circle, radius r, at speed v: $F = \frac{mv^2}{r}$ MaT
- This force is provided by the gravity, so we need $\frac{mv^2}{r} = \frac{GMm}{r^2}$, or $v = \sqrt{\frac{GM}{r}}$
- If the stars are faster than this, they will fly out. If they are slower, they will fall in

Bigl

Dark Matter

- Assume that all the mass in a galaxy is in the stars, which we can see
- The gravity holding a star in orbit at speed v a distance r from the center comes from the total mass of all the stars inside that orbit (a sphere is equivalent to a single point of mass at the center)
- By measuring how many stars there are at inside each ring of the galaxy, we know how strong the gravity should be, and therefore how fast the stars can orbit: $v = \sqrt{\frac{GM}{r}}$ MaT
- Stars in the outer parts of the galaxy should orbit more slowly than those closer to the center

Vera Rubin (1975) working with a new sensitive instrument measuring the velocity of stars at the edges of galaxies. Stars at the edge of galaxies orbit faster than those closer to the center!



Need extra mass in a galaxy that we cannot see!

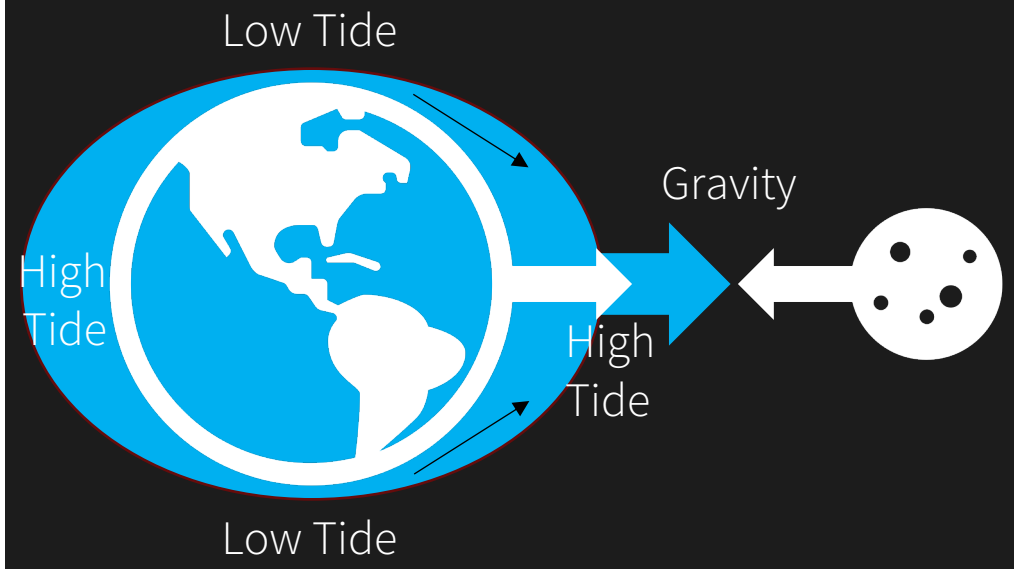
This is dark matter – a mysterious form of mater that feels the force of gravity, but does not emit or interact with any light

There's about 5x more dark matter than regular matter (that you, me, planets, stars are made of)

Bigl

Tides

- The gravitational force from the Earth pulls the moon towards it
- The gravitational force from the moon pulls the Earth towards it
- The moon pulling on the oceans gives us tides
  - High tide on side of Earth near moon as pull is strongest closest to moon
  - Low tides on the sides 90° to moon (water pulled away)
  - The pull on the Earth is stronger than the pull on the water on the far side, so Earth moves slightly towards moon, giving high tide on far side from moon



The Equivalence Principle

All objects experience the same acceleration under gravity, independent of mass, so if you are freely falling alongside an object, you don't notice its gravitational acceleration.

In a freely falling (non-rotational) frame (or to a freely-falling observer), the laws of physics are consistent with there being no gravity

(we can always 'transform the gravity away' or find an observer who doesn't feel it)

Equivalence principle has been tested to better than 1 part in 10^{14}

- Torsion balance, measuring acceleration of different masses towards the Earth, Sun and Galactic center
- Orbits of pulsars (see later)
- Atom interferometry (Stanford), measuring acceleration of different atoms

QaD

- Motion of the planets and stars in a galaxy

MaT

- Kepler’s Law
- Inverse square laws

WILtY

- Planets follow elliptical orbits around the Sun

BigI

- Newton’s laws of motion
- Mass, force & momentum
- Universal Law of Gravitation
- Dark matter
- Tides
- The equivalence principle

“We have peered into a new world and have seen that it is more mysterious and more complex than we had imagined.
Still more mysteries of the Universe remain hidden.
Their discovery awaits the adventurous scientists of the future —I like it this way.”
– Vera Rubin